

**Philadelphia University**  
**Faculty of Science**  
**Department of Basic Sciences and Mathematics**  
Real Analysis                      Second Exam

Student name: \_\_\_\_\_

Number: \_\_\_\_\_

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1) State Monotone Convergent Theorem

2) Determine whether the following sequences converge or diverge

a)  $\left(\frac{3+\cos n}{n^2}\right)$ .

b)  $\left(\cos\left(n\frac{\pi}{2}\right)\right)$

c)  $\left(\frac{n3^n}{2^{2n+1}}\right)$



3) Prove that every convergent sequence is Cauchy sequence.

4) Prove that a bounded sequence of real numbers has a convergent subsequence.

5) Let  $x_1 > 1$  and  $x_{n+1} = 2 - \frac{1}{x_n}$  for  $n \in \mathbb{N}$ . Show that  $(x_n)$  is decreasing, bounded below by 1, then find the limit. ( Hint: let  $x_1 = 2$  )



6) Determine whether the following statements are true or false, **justify**

a) Every convergent sequence is monotone

b) If  $(x_n)$ ,  $(y_n)$  are divergent sequences, then  $(x_n + y_n)$  is divergent

c) If  $\lim_{n \rightarrow \infty} |x_{n+1} - x_n| = 0$ , then  $(x_n)$  converge